STAT C-202 ALGEBRA

Algebra (202)

1. Solve the equation $3x^4 - 40x^3 + 130x^2 - 120x + 27 = 0$ given that the product of two of its roots is equal to the product of the other two.
2. Solve the equation
$$3x^4 - 8x^3 + 21\,x^2 - 20x + 5 = 0$$
given that the sum of two of its roots is equal to the sum of the other two.
3. (a)Solve the equation $x^3 - 5x^2 - 16x + 80 = 0$, the sum of two of its roots being zero.
(b)Solve the equation $2x^3 - x^2 - 22x - 24 = 0$ two of the roots being in the ratio of 3 :4.

4. Solve the equation
$x^4 - 12x^3 + 49x^2 - 78x + 40 = 0$ by removing its second term.

5. If α, β, γ are the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$ form an equation whose roots are $\beta^2 + \gamma^2$, $\gamma^2 + \alpha^2$, $\alpha^2 + \beta^2$.
6. (a) If α, β, γ be the roots (all non zeros) of the equation $x^3 - px^2 + qx - r = 0$ find the value of (i) $(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$, (ii) $\sum(\alpha / \beta)$
(b) Form a cubic whose roots are the values of α, β, γ given by the relations $\sum \alpha = 3, \sum \alpha^2 = 5, \sum \alpha^3 = 11.$ Hence find the value of $\sum \alpha^4$.

7.. If α,β, γ are the roots of the cubic $x^3 + px^2 + qx + r = 0$ form the equation whose roots are
$\frac{\alpha}{\beta+\gamma-\alpha}, \frac{\beta}{\gamma+\alpha-\beta}, \frac{\gamma}{\alpha+\beta-\gamma}$.

8. Solve the equation $x^3 + 6x^2 + 12x - 19 = 0$ by removing its second term.

9. (i) Find the sum of the fifth powers of α, β, γ – the roots of the equation $x^3 - x + 1 = 0$.
(ii) Find the sum of the reciprocals of the fifth powers of α, β, γ – the roots of the equations$x^3 + 2x^2 + 1 = 0$.
10. Show that a skew-symmetric determinant of order 4 is the square of the polynomial function of its elements.

11. Express
$$\begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ac - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix}$$
as the square of a determinant and hence find its value. If the value of the given determinant is zero, then prove that $a^3 + b^3 + c^3 = 3abc$.
12. Express
$$\begin{vmatrix} (1+ax)^2 & (1+ay)^2 & (1+az)^2 \\ (1+bx)^2 & (1+by)^2 & (1+bz)^2 \\ (1+cx)^2 & (1+cy)^2 & (1+cz)^2 \end{vmatrix}.$$
as product of two determinants and hence evaluate it.

13. When is a rectangular matrix said to be in reduced echelon form? Transform the following matrices into reduced echelon form and circle the pivot positions in the final matrix:

$$A=\begin{bmatrix}1 & 2 & 3 & 4\\ 4 & 5 & 6 & 7\\ 6 & 7 & 8 & 9\end{bmatrix}. \qquad B=\begin{vmatrix}2 & 1 & 0 & 5\\ 3 & 2 & 7 & 9\\ 1 & 4 & 9 & 3\end{vmatrix}$$

14. When is a matrix said to be in
    (i) Echelon form,
    (ii) Reduced echelon form?

15.(a) Show that the system of simultaneous equations

$$x-y+2z = 4$$

$$3x+y+4z = 6$$

$$x+y+z = 1$$ are consistent and hence solve them.

(b) For what values of λ the equations

$$x+y+z=1$$
$$x+2y+4z=\lambda$$
$$x+4y+10z=\lambda^2,$$ have a solution and solve the equations

16. Investigate for which values of θ and μ the system of equations

$$x+2y+3z=5,$$
$$3x-y+2z=1,$$
$$3x-y+\theta z=\mu$$

will have
(i) No solution,
(ii) A unique solution,
(iii) An infinite number of solutions.

17. Solve with the help of Cramer's rule

$$ax+by+cz=k,$$
$$a^2x+b^2y+c^2z=k^2,$$
$$a^3x+b^3y+c^3y=k^3.$$

18. Define a circulant determinant. Show that

$$\begin{vmatrix}a & b & c & d\\ d & a & b & c\\ c & d & a & b\\ b & c & d & a\end{vmatrix}$$

has $a+b+c\lambda^2+d\lambda^3$ as a factor were λ is a root of $x^4=1$. Hence show that the determinant is equal to $(a+b+c+d)(a-b+c-d)\{(a-c)^2+(b-d)^2\}$.

19. Solve

$$\Delta = \begin{vmatrix}x-2 & 2x-3 & 3x-4\\ x-4 & 2x-9 & 3x-16\\ x-8 & 2x-27 & 3x-64\end{vmatrix} = 0$$

20.(a) Prove that a skew-symmetric determinant of odd order vanishes.

(b) Find the area of the parallelogram whose vertices are

$$(-2,\ -2),(0,\ 3),(4,\ -1),(6,\ 4)$$

21.(a) If $A=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$ prove that $(aI+bA)^n=a^nI+na^{n-1}bA$ where I is the 2 rowed unit matrix, n is a positive integer and a , b are arbitrary scalars.

(b) Show that the possible square roots of the two rowed unit matrix I are $\pm I$ and $\begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}$ where, $1-\alpha^2=\beta\gamma$.

22. (a) If B, C are n x n matrices and if A = B + C then prove that

$A^{P+1} = B^P [B + (P+1)C]$ provided B and C commute,

$C^2 = 0$ and P is a positive integer.

(b) If A is a symmetric and B is a skew-symmetric matrix, both of order n such that (A+B) is non singular and $C = (A+B)^{-1} \cdot (A-B)$, then prove that

(a) $C'(A+B)C = A+B$

(b) $C'(A-B)C = A-B$

(c) $C'AC = A$

23. Define Orthogonal and Unitary matrices. If A is a square matrix and $A-\frac{1}{2}I$ $A+\frac{1}{2}I$ are orthogonal (I is an identity matrix of order same as A), prove that A is a skew symmetric and $A^2 = -\frac{3}{4}I$. Deduce that A is of even order.

24.(a) If A and B are n-square matrices, then prove that adj(AB) =adj B.adj A

(b) Find the value of adj(P^{-1}) in terms of P where P is a non singular matrix & hence show that adj ($Q^{-1}BP^{-1}$)=PAQ given that adj B=A & |P|=|Q|=1

25. Let e be the column vector with elements (1,1,….,1) & e' its transposed row vector. Let A be n- square matrix & I the unit matrix .Let M(x) be given by $M(x)= I + xAee'$,x is a scalar .Prove that M(x)M(y)=M(x+y+kxy) where K is the scalar $e'Ae$ .Verify that reciprocal of M(x) is M(-x/(1+kx)).

26. Prove that every Hermitian matrix H can be uniquely expressed as $P+iQ$ where P and Q are real symmetric and real skew symmetric matrices respectively. Further, show that $H^\theta H$ is real iff $PQ=-QP$.

27.If F (α)= $\begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$;

G (β)= $\begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix}$ Show that the inverse of the matrix F (α) G (β) is G (-β) F (G (-β) F (-α).

28. For every real number x such that -1<x<1, let A(x) be the matrix defined as

$$A(x) = (1-x^2)^{-1/2}\begin{pmatrix} 1 & -x \\ -x & 1 \end{pmatrix}$$

Show that

$$A(x)A(y) = A(z), \text{ where } z = \frac{x+y}{1+xy}$$

Deduce that $[A(x)]^{-1} = A(-x)$.

29. (a) Define elementary matrices. Show that elementary matrices are non-singular. Obtain their inverses.

(b) Express $A=\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ as a product ofelementary matrices.

30. Define the rank of a given matrix. Prove that the rank of the product of two matrices can not exceed the rank of either matrix.

31 Obtain the rank of the following matrix

$$\begin{bmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{bmatrix}$$

32. If no two of a,b,c are equal and no two of p, q, r are equal, show that the matrix

$$A = \begin{pmatrix} 1 & a & p & ap \\ 1 & b & q & bq \\ 1 & c & r & cr \end{pmatrix}$$

is of rank 3.

33. If a, b and c are all unequal, find using only row operations, the rank of the matrix

Type equation here.
$$\begin{pmatrix} 0 & b-a & c-a & b+c \\ a-b & 0 & c-b & c+a \\ a-c & b-c & 0 & a+b \\ b+c & c+a & a+b & 0 \end{pmatrix}$$

34. Let A be an nxn nonsingular matrix. Compute the inverse of A by method of partitioning and hence obtain the inverse of $\begin{bmatrix} I & Q \\ O & R \end{bmatrix}$ where R^{-1} exists and is known. I is the identity sub matrix.

35. Compute the inverse of partitioned matrix

$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$ of order $n \times n$,

assuming A_{22} is a nonsingular sub matrix of order $s \times s$.

36. Compute the inverse of Partitioned matrix

$A = \begin{bmatrix} B & 0 \\ D & 1 \end{bmatrix}$ where B is n x n , D is 1 x n ,

0 is n x 1, 1 s 1 x 1

37. Given the existence of all necessary inverses, show that inverse of

$$\begin{bmatrix} I & P \\ Q & I \end{bmatrix} = \begin{bmatrix} (I-PQ)^{-1} & -(I-PQ)^{-1}P \\ -Q(I-PQ)^{-1} & I+Q(I-PQ)^{-1}P \end{bmatrix}$$

38. Find XGX', where G is a generalized inverse of $X'X$

(a) Let $X = \begin{pmatrix} 1 & -1 & -2 \\ 1 & -1 & -2 \\ 1 & -1 & -2 \end{pmatrix}$

(b) Let $X = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$.

39. Discuss the algorithm for finding a generalized inverse of a given matrix. How will you find a symmetric generalized inverse for a symmetric matrix of order n.

40. Prove, when G is generalized inverse of $X'X$, that

(i) $G'$is also generalized inverse of $X'X$,

(ii)$XGX'X = X$,

(iii) XGX' is invariant of G.,

(iv) XGX' is symmetric whether G is or not.

41.(a) If the characteristic roots of A are$\lambda_1, \lambda_2, \ldots, \lambda_n$, then show that characteristic roots of A^2 are $\lambda_1^2, \lambda_2^2, \ldots, \lambda_n^2$. Further, if A is an idempotent matrix, show that its roots are zero or unity.

(b) Prove that the modulus of each characteristic root of a unitary matrix is unity.

42. If the characteristic equation of 3x3 matrix A be $\lambda^3 - p\lambda^2 + q\lambda + r = 0$ prove that the characteristic equation of adj A is $\lambda^3 - q\lambda^2 - rp\lambda - r^2 = 0$

43 Prove that the characteristic roots of a square matrix A of order 3 are same as that of any of its transformed matrix, PAP^{-1} where P is any non-singular matrix of order 3. Also if

$$P=\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}; \quad A=\frac{1}{2}\begin{pmatrix} b+c & c-a & b-a \\ c-b & c+a & a-b \\ b-c & a-c & a+b \end{pmatrix}$$

Determine the characteristic roots of the matrix A.

44. Prove that any two characteristic vectors corresponding to two distinct characteristic roots of a Hermitian matrix are orthogonal

45.(a) Find the characteristic roots of the matrix

$$A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & -3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

and show that the characteristic vectors associated with its distinct characteristic roots are mutually orthogonal.

(b) Find the characteristic roots of the matrix $A=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and show that the characteristic vectors associated with its distinct characteristic roots are mutually orthogonal.

46. State Cayley-Hamilton theorem. Given$A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$, express $A^4 - 4A^3 - A^2 + 2A - 5I$ as a llinear polynomial in A and hence evaluate it.

47. The characteristic roots of a square matrix of order 3 are 1,-1 ,2. Express A^6 as the quadratic polynomial in A.

48. Compute $2A^8-3A^5+A^4+A^2-4I$, where A is the matrix

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

49. Reduce the real quadratic form $3x^2 - 3y^2 - 5z^2 - 2xy - 6yz - 6zx$ to its canonical form and find its rank, signature and index.

50. Identify the nature of the quadratic forms

(a) $21x_1^2 + 11x_2^2 + 2x_3^2 - 30x_1x_2 - 8x_2x_3 + 12x_3x_1$

(b) $x_1^2 + 6x_2^2 + 18x_3^2 + 4x_1x_2 + 8x_1x_3 - 4x_2x_3$

(c) $4x^2 + 9y^2 + 2z^2 + 8yz + 6zx + 6xy$

(d) $x_1^2 + 6x_2^2 + 18x_3^2 + 4x_1x_2 + 8x_1x_3 + 4x_2x_3$

(e) $6x_1^2 + 3x_2^2 + 14x_3^2 + 4x_2x_3 + 18x_3x_1 + 4x_1x_2$

and hence find the rank, index and signature of the form.

STAT C- 201
PROBABILITY AND PROBABILITY DISTRIBUTION

(b) If X and Y are independent Cauchy variates with parameters (λ_1, μ_1) and (λ_2, μ_2) respectively then find the distribution of $(X + Y)$. 1

(c) If the characteristic function $\varphi_x(t)$ of a continuous random variable X is given, then how will you find p.d.f $f(x)$? 1

(d) State the relationship between $M_x(t)$ and μ'_r. 1

(e) If two variables X and Y are independent then what is E (Y/X) ? 1

(f) Let X be a random variable with probability distribution :

X = x	–2	2	4
P (X = x)	1/8	½	3/8

Find E(X) and Var (X) 2

(g) Let X and Y have the joint p.d.f.

$f(x, y) = c\,(2x + y)\ ;\ 0 \le x \le 1,\ 0 \le y \le 2,$

then find c. 2

(h) If Cov $(aX + bY, bX + aY) = ab \text{ Var } (X + Y)$, then comment on the independence of X and Y. 2

(i) The probability distribution of a random variable X is :

$P(X = x) = (1/2)^x$; $x = 1, 2, \ldots\ldots$

Then find mean and mode of X. 2

(j) If the p.d.f. of a random variable X is :

$f(x) = c \exp[-(x^2 - 6x + 9)/24]$; $-\infty < x < \infty$,

then find c, mean and variance of X. 2

Section – A

2. (a) Let X be a random variable with p.m.f. given by : 6

X = x	0	1	2	3	4	5	6	7
P(X = x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

(i) find k,

(ii) find $P(X < 6)$ and $P(0 < X < 5)$,

P.T.O.

(iii) if $P(X \leq a) > ½$, find the minimum value of a,

(iv) determine the distribution function of X.

(b) For the distribution :

$$dF(x) = \begin{cases} y_0\left(1 - \frac{1}{a}|x - b|\right)dx & , \quad b - a < x < b + a \\ 0 & , \quad \text{otherwise} \end{cases}$$

calculate y_0, mean and variance. 6

3. (a) Let X be a random variable taking non-negative integral values. If the moments of X are given by : 6

$E(X^r) = 0.6$; r = 1, 2, 3.........

then find m.g.f. of X. Also, show that $P(X = 0) = 0.4$, $P(X = 1) = 0.6$, $P(X \geq 2) = 0$.

(b) An urn contains balls numbered 1, 2, 3. First a ball is drawn from the urn and then a fair coin is tossed as many times, as the number shown on the ball drawn. Find the expected number of tails. 6

4. (a) Define the characteristics function of a random variable. Show that the characteristic function of sum of two independent random variables is equal to the product of their characteristic functions. Is the converse true, if not justify. 6

(b) If the joint p.m.f. of X and Y is given by : 6

$$p(x, y) = \frac{(x-y)^2}{7} \quad \text{for } x = 1, 2 \text{ and } y = 1, 2, 3.$$

find :

(i) the marginal distribution of Y,

(ii) the joint distribution of U = X + Y and V = X – Y,

(iii) the marginal distribution of U.

Section – B

5. (a) Find the m.g.f. of standard binomial variate $(X - np)/\sqrt{npq}$ and obtain its limiting form as $n \to \infty$. Also interpret the result. 6

(b) Let X be the negative binomial variate with p.m.f.,

$$f(x) = \begin{cases} \binom{k+x-1}{x} q^x p^k & ; \quad x = 0, 1, 2, \ldots. \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Show that the moment recurrence formula is :

$$\mu_{r+1} = q\left(\frac{d\mu_r}{dq} + \frac{rk}{p^2}\mu_{r-1}\right), \ r = 1, 2, \ldots\ldots$$

Hence find variance of X. 6

6. (a) If X and Y are independent Poisson variates such that

$V(X + Y) = 4$ and $P(X = 3 \mid X + Y = 6) = 5/16$,

then find E(X) and V(Y). 6

(b) If $X \sim N(\mu, \sigma^2)$, then find the p.d.f. of $Y = e^x$ and identify it. Also find the coefficient of variation of Y. 6

7. (a) If $X \sim N(0, 1)$ and $Y \sim N(0, 1)$ are independent random variables, then find the mean deviation about mean of $(X - Y)$. 6

(b) Let $X \sim \beta_1(m, n)$ and $Y \sim \gamma(m + n)$ be independent random variables, $(m, n>0)$. Find the p.d.f. of XY and identify the distribution. 6

8. (a) Define hypergeometric distribution and find its mean. Obtain binomial distribution as a limiting case of hypergeometric distribution. 6

(b) $X_1, X_2, \ldots\ldots X_n$ are independent random variable having exponential distribution each with parameter λ. Obtain the distribution of $Y = X_1 + X_2 + \ldots\ldots + X_n$ and hence find mean of Y. 6

(iv) Mean deviation is least when taken from

________ .

(v) $M_X(t) = M_Y(t)$ implies ______

(b) State the conditions under which :

(i) binomial distribution tends to normal distribution

(ii) negative binomial distribution tends to Poisson distribution

(c) If X and Y are two independent random variables such that

$E(X) = \lambda_1$, $V(X) = \sigma_1^2, E(Y) = \lambda_2, V(Y) = \sigma_2^2$

then prove that $V(XY) = \sigma_1^2 \sigma_2^2 + \lambda_1^2 \sigma_2^2 + \lambda_2^2 \sigma_1^2$

(d) If X and Y are independent normal variates with means 6, 7 and variances 9, 16 respectively. What is the distribution of $4x - 3y$?

(e) A Poisson distribution has a double mode at $x = 1$ and $x = 2$. What is the mean of the distribution ?

(f) If $X \sim B(n, p)$, what is the distribution of $Y = n - X$? 5,2,2,2,2,2

Section I

2. (a) Show that, for the following distribution :

$$f(x)=\frac{2a}{\pi}\left(\frac{1}{a^2+x^2}\right), -a \le x \le a$$

$$\mu_2=\frac{a^2(4-\pi)}{\pi} \text{ and } \mu_4=a^4\left(1-\frac{8}{3\pi}\right).$$

(b) Let X be a continuous random variable with p.d.f. $f(x)$. Let $Y = X^2$. Show that the random variable Y has p.d.f. given by :

$$g(y)=\begin{cases}\frac{1}{2\sqrt{y}}[f(\sqrt{y})+f(-\sqrt{y})] & , y>0 \\ 0 & , y \le 0\end{cases}$$ 6,6

3. (a) Define mathematical expectation of a random variable. For two random variables X and Y, prove that

E (X + Y) = E (X) + E(Y)

provided all the expectations exist.

(b) A deck of n numbered cards is thoroughly shuffled and the cards are inserted into n numbered cells one by one. If the card number 'i' falls in the cell 'i', we count it as a match, otherwise not. Find the mean and variance of total number of such matches. 6,6

4. (a) Define the characteristic function $Q_X(t)$ of a random variable X. If p.d.f. of X is symmetric about origin, then prove that $Q_X(t)$ is real and even valued function of t. Name one distribution for which this result holds.

(b) Let the random variable X assume the value r with the probability law

$$P(X=r)=q^{r-1}\,p, r=1,2,3....$$

Find the m.g.f. of X and hence its mean and variance. 6,6

Section -II

5. (a) Obtain mean deviation about mean of binomial distribution with parameters n and p.

(b) If X is a Poisson variate with parameter m and μ_r is the r^{th} central moment, prove that

$$m\left[\binom{r}{1}\mu_{r-1}+\binom{r}{2}\mu_{r-2}+.....+\binom{r}{r}\mu_0\right]=\mu_{r+1}.$$ 6,6

6. (a) Show that for the negative binomial $(Q-P)^{-r}$ where $Q-P=1$, cumulant generating function $K(t)=-r\log[1-P(e^t-1)]$. Hence deduce that

$K_1 = rP, K_2 = rPQ$. Also show that mean is less than variance for negative binomial distribution.

(b) Obtain binomial distribution as a limiting case of Hypergeometric distribution. 6,6

7. (a) If X $\sim$ N (6,9) and Y $\sim$ N(7,16) are independent determine λ such that

$P(2X + Y \leq \lambda) = P(4X - 3Y \geq 4\lambda)$.

(b) Obtain characteristic function of standard Laplace distribution. Hence find β_1 and β_2. 6,6

8. (a) Show that the mean value of positive square root of a $\gamma(\mu)$ variate is $\Gamma\left(\mu + \frac{1}{2}\right) / \Gamma(\mu)$. Hence prove that the mean deviation of a normal variate from its mean is $\sqrt{\frac{2}{\pi}}\sigma$, where σ is standard deviation of the distribution.

(b) "The role of Cauchy distribution often lies in providing counter examples". Justify. 6,6

1. (a) For what value of k,

$$F(x) = \begin{cases} k\left(1-e^{-x}\right)^2 & ; \quad x > 0 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

is a c.d.f. (1)

(b) Verify whether $p(x) = \frac{2x}{k(k+1)}$; x = 1,2, ... k is the p.m.f. of a random variable x. (1

(c) State the relationship between $E(X^r)$ and $Ø_x(t)$. (1

P.T.(

(d) If X follows beta distribution of second kind then what transformation of X follows beta distribution of first kind. (1)

(e) For what value of c, $E(X-c)^2$ is minimum. (1)

(f) Let X and Y have the joint p.d.f. given by

$f(x, y) = 4xy; \quad 0 < x < 1; 0 < y < 1.$

Find $E(Y|X = 1/2)$. (2)

(g) If $X \sim B(n, p)$ and $P(X \leq 2) = P(X \geq n-2)$, then find p. (2)

(h) Identify the variate whose m.g.f. is given by

$$M_X(t) = \frac{(1+2e^t)^3}{27} e^3(e^t - 1)$$ (2)

(i) If Var(X) = Var(Y) and 2X + Y and X – Y are independent then check the independence of X & Y. (2)

(j) If $X \sim N(0, 1)$, then find the variance of X^2. (2)

SECTION A

2. (a) Let X be a random variable with c.d.f. given by :

$$F(x) = \begin{cases} 0 & ; \quad x < -1 \\ \frac{1}{4} & ; \quad -1 \leq x < 1 \\ \frac{1}{2} & ; \quad 1 \leq x < 3 \\ \frac{3}{4} & ; \quad 3 \leq x < 5 \\ 1 & ; \quad x \geq 5 \end{cases}$$

Find (i) $P(X \leq 3)$ (ii) $P(X = 3)$ (iii) $P(X \geq 1)$ (iv) $P(-0.4 < X < 4)$ (v) p.m.f. of X.

(b) Let X be a random variable with p.d.f. :

$$dF(x) = \frac{x}{b^2} e^{\frac{-x^2}{2b^2}} dx \quad ; \quad x \geq 0.$$

Find Q_1, Q_3 and standard deviation. Also show that the ratio of distance between the quartiles to the standard deviation is independent of the parameter b.

(6,6)

3. (a) Let X be a random variable with p.d.f.

$$f(x) = \begin{cases} \frac{1}{6} & ; \quad -3 \leq x \leq 3 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

Find the p.d.f. of $Y = 2X^2 - 3$.

(b) Ten cards are drawn at random from hundred cards numbered 1 to 100. Find the expectation of sum of points on the cards drawn. (6,6)

4. (a) If X and Y are two independent random variables with p.d.f.

$$f(x) = e^{-x} \; ; \; x \geq 0 \text{ and } g(y) = 3e^{-3y}; \; y \geq 0, \text{ then}$$

find the p.d.f. of $Z = X/Y$ and identify the distribution.

(b) (i) Define cumulant generating function. Discuss the effect of change of origin and scale on cumulant generating function.

(ii) If X is a random variable having cumulants k_r given by

$$k_r = (r-1)! \; p \, a^{-r} \; ; \quad p > 0, \; a > 0, \; r = 1, 2, \ldots,$$

then find the moment generating function of X. (6,3+3)

SECTION B

5. (a) Find mean deviation about mean of a binomial distribution with parameters n and p.

(b) Let X and Y be independent Poisson variates with

$$V(X+Y) = 6 \text{ and } P(X = 2 \mid X + Y = 4) = 3/8.$$

Find the standard deviation of X and Y. (6,6)

6. (a) Derive moment generating function of negative binomial distribution and hence find mean and variance of the distribution.

(b) If a random variable X has ~~log~~normal distribution with parameters μ and σ, then find mean, median and mode of X. (6,6)

7. (a) Find first four moments, β_1 and β_2 of a standard Laplace distribution. Also interpret the result.

(b) (i) Show that for the normal distribution $N(\mu, \sigma^2)$, the points of inflexion are equidistant from mean.

(ii) "The role of Cauchy distribution often lies in providing counter examples". Justify. (6,3+3)

8. (a) Define 'lack of memory' property. Show that geometric distribution lacks memory.

(b) If X is a gamma variate with parameter λ then obtain its m.g.f. Hence deduce that the m.g.f. of standard gamma variate tends to $\exp\left(\frac{t^2}{2}\right)$ as $\lambda \to \infty$. Also interpret the result. (6,6)

...pt 5 more questions by selecting at least *two* questions from each Section.

...ill in the blanks :

...) If X and Y are independent binomial variates with parameters $\left(6, \frac{1}{3}\right)$ and $\left(12, \frac{1}{3}\right)$ respectively, then the distribution of X + Y is ....

...i) The distribution for which $\phi(t) = e^{-|t|}$ is ....

...ii) If X ~ N(3, 25), then median of the distribution is ....

...v) A random variable X has probability function $f(x) = \frac{1}{2^x}$, $x = 1, 2, 3, \ldots$ then the Mode of this distribution is ....

...v) If V(X) = 2, then V(2 – X) is ....

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(b) State the conditions under which Poisson distribution is a limiting case of binomial distribution.

(c) Let the p.d.f. of a normal variate be $f(x) = Ce^{-\frac{1}{4}x^2 + x}$. Find C and E(X).

(d) Identify the distribution whose m.g.f. is $M_x(t) = 4(3e^{-t} - 1)^{-2}$.

(e) The experiment is to toss two balls in 5 boxes in such a way that each ball is equally likely to fall in any box. Let X denotes the number of balls in the fifth box. Find the probability distribution of X.

(f) State the relation between first four central moments and cumulants. 1×5,2,2,2,2,2,

Section I

2\. (a) For the given probability function :

$$f(x) = y_0 x(2 - x); \quad 0 \le x \le 2.$$

Find mean, variance, μ_{2n+1}, median, mode, β_1 and β_2. Is the distribution symmetric ?

(b) If μ_r' is moment of rth order about the origin and k_j is the cumulant of jth order, prove that : 6,6

$$\frac{\partial \mu_r'}{\partial k_j} = \binom{r-1}{j-1} \mu_{r-j}'.$$

3\. (a) A continuous r.v. X has the distribution function $F(x) = \begin{cases} 0, & \text{if } x \le 1 \\ k(x-1)^4, & \text{if } 1 < x \le 3 \\ 1, & \text{if } x > 3 \end{cases}$

Find :

(i) k

(ii) p.d.f. of x and

(iii) $p\left(X < \frac{2}{3} \middle| X > \frac{1}{3}\right)$.

(b) The p.d.f. of a random variable X is $f(x) = \frac{1}{6}$, $-3 \le x \le 3$. Find the p.d.f. of $Y = 4X^2 - 3$ and verify the result. 6,6

4. (a) A box contains a white and b black balls. c balls are drawn at random. Find the expected value of the number of white balls drawn.

(b) Define the characteristic function of a random variable. Show that the characteristic function of sum of two independent random variables is equal to the product of their characteristic functions.

Justify that, the converse of the above statement need not be true. 6,6

Section II

5. (a) If p is the constant probability of success at any single trial, obtain the probability of x successes out of n independent trials. Also find the mode of the resulting distribution.

(b) If $X \sim N(\mu, \sigma^2)$, obtain the p.d.f. of $U = \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2$ and identify the distribution. 6,6

6. (a) X is a negative binomial variate with p.m.f. :

$$f(x) = \begin{cases} \binom{k+x-1}{x} q^x p^k, & x = 0, 1, 2, \ldots\ldots \\ 0, & \text{otherwise} \end{cases}$$

Show that the moment recurrence formula is $\mu_{r+1} = q\left(\frac{d\mu_r}{dq} + \frac{rk}{p^2}\mu_{r-1}\right)$ and hence compute μ_2 and μ_3.

(b) Find the characteristic function of standard Laplace distribution and hence, find its mean and standard deviation. 6,6

P.T.O.

(b) Obtain the m.g.f. of a random variable X having p.d.f.

$$f(x) = \begin{cases} x & ; 0 \le x < 1 \\ 2 - x & ; 1 \le x \le 2 \\ 0 & ; \text{elsewhere} \end{cases}$$

Determine first four moments about mean. (6,6)

SECTION II

6. (a) Find the mode of a binomial distribution with parameter n and p. Hence find mode of $X \sim B(8, 1/3)$.

(b) If X is a Poisson variate with mean λ, show that $E(X^2) = \lambda E(X+1)$.

Also show that if $\lambda = 1$, $E|X - 1| = 2/e$. (6,6)

7. (a) If X is a negative binomial variate with p.m.f. given by

$$p(x) = \binom{k+x-1}{x} q^x p^k, \; x = 0, 1, 2, \ldots$$

Show that the recurrence relation for moments is :

$$\mu_{r+1} = q\left(\frac{d\mu_r}{dq} + \frac{rk}{p^2}\mu_{r-1}\right), \; r = 1, 2, 3, \ldots$$

Hence find Var(X).

(b) Find mean deviation about mean of a two parameter Laplace distribution. (6,6)

8. (a) If $X \sim$ Cauchy distribution (λ, μ), then show that

$1/X \sim$ Cauchy distribution $(\lambda/(\lambda^2 + \mu^2), \mu/(\lambda^2 + \mu^2))$.

(b) Write the equation of a normal probability curve with mean μ and standard deviation σ. State eight main characteristics of the normal probability curve. (6,6)